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SOUND FIELDS : RADIATED BY ARRAYED MULTIPLE SOUND SOURCES

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0. INTRODUCTION

The recent trend to increase both the level and the coverage of the acoustic radiated power, leads to an inflation of the number of individual sound sources (boxes). Nowadays some concerts involve more than one hundred boxes. Unfortunately arraying boxes arises interference phenomena which are not easily mastered. As far as linear frequency response, directivity control and power level at long distance are concerned, the results can be disastrous.

Large arrays of boxes, the characteristics of which are well known, lead most of the time to very surprising results. The question is to know whether or not it is possible to predict the behaviour of an array when the behaviour of each element is known.

Our purpose is to describe the sound field produced by arrays in such a way that criteria for arrayability can be defined.

The sound field of an ensemble of sources can be roughly divided into three categories : near, transition, and far region. The terms near and far are usually defined as being related to the distance between the listener and the array. In fact the border lines are highly depending on frequency. The geometry depending on the conditions the far region can be only a few meters away from the arrayed sources or hundreds of meters. We have chosen, for convenience to rename regions : the near region will be named «Fresnel». It may be approximated quite correctly ; the transition region will be named «chaotic» because it comprises many patterns and is not amenable to simple descriptions ; the far region will be named the «Fraunhofer» region. It can be described, despite its numerous interference patterns.

We will first define simple ways to know how these three regions are distributed in the hall as a function of the arrayed sound sources geometry and as a function of the sound frequency.

Then we will describe the average behaviour of the sound field inside each of them.

We limit the scope of this paper to two kinds of elementary sound sources : circular and rectangular, flat isophase pistons.

The object of this paper is concerned only with large halls or open air installations and thus we will examine the sound fields for listening distances larger than the size of the array. Room and floor reflexions will not be taken into account.

I. SINGLE SOURCE

I. 1. Generalities

In order to focus on the important aspects we omit, most of the time, unnecessary multiplicative constants.

We use the Kirchoff formula which gives the sound pressure at a point M inside a volume as a surface integral. () : [1]

$$P(M) \propto k \int_{\text{surface}} u(s) \frac{e^{ikr}}{r} \frac{1 + \cos \Psi}{2} ds \quad (1)$$

k is the wave number : $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c}$

r is the distance from the surface element ds to M.

f is the frequency and c the sound velocity assumed to be 330 m/s.

u(S) is the normal velocity of the surface element ds and Ψ is the angle between the normal to the surface and the vector \vec{r} .

Figure 1 displays the situation in the case of a plane vibrating piston. The surface is the total plane with u(S) non zero only on the piston.

figure 1

The term $\frac{1 + \cos \Psi}{2}$ can be interpreted as the sum of a monopole and a dipole. As we

will be interested only by conditions such that $\Psi \leq 45^\circ$, we will neglect the $\cos \Psi$ variations in the qualitative approach but we keep this factor in our numerical integration program. We thus assume in the following that

$$\frac{1 + \cos \Psi}{2} \approx 1$$

In equation (1), the product $ku(S)ds$ is proportional to $j\omega u(S)ds$ and since we regard only pure frequency sound with $e^{i\omega t}$ time dependence, this product is also proportional to the time derivative of $u(S)ds$. We assume that elementary sound sources are of the constant volume acceleration type, so that $ku(S)ds \propto a_0 ds$ where a_0 is a constant acceleration. This can lead to the simplified expression :

$$P(M) \propto \int_{\text{surface}} a_0(s) \frac{e^{ikr}}{r} ds \quad (2)$$

The phenomena being complex we will look at them from several points of view, namely : analytic simplified integration (when possible), Fresnel zones and geometric diffraction. The numerical exact integration program is used to verify our qualitative results

but this is only interesting when the answers are already partly known.

The numerical integration is of little interest to get insights into these phenomena.

One of the concepts which is widely used in characterising sound sources, is the directivity [2]. There are several definitions of the directivity, but the basic idea is to give a number for the solid angle extent of the flow of energy of a given sound source within a 6dB max deviation.

An isotropic point-source radiates over 4π steradians and a directive source radiates most of its energy into a solid angle $\Delta\Omega$. The directivity factor Q is defined as :

$$Q = \frac{4\pi}{\Delta\Omega}$$

The interest of this rough number is to allow the determination of critical distances for intelligibility.

This is shown in fig. 2.

figure 2

S being the area of a hall, a being the average absorption coefficient of the walls, and d the distance from the sound source.

The critical distance d_c is then given when $a \ll 1$ by :

$$d_c = 0.14 \sqrt{Q S a}$$

The directivity concept, by itself, assumes that the source is a point and therefore that the energy flow expands radially. We will show that inside a Fresnel region the average energy flow is either constant or proportional to $\frac{1}{r}$. It is only in the Fraunhofer region that the usual directivity concept makes sense again.

A very common way to display data is polar plots. Clearly again, this is valid only in the Fraunhofer region. Polar plots in a plane wave are useless to describe the sound intensity. However this is what occurs when the microphone stands inside the Fresnel region of a plane array !

I. 2. Single circular source

We describe here the sound field created by a single flat vibrating piston on a plane infinite wall. Let D be the diameter of the piston, θ the angle of observation with respect to the normal axis.

I. 2. a – The Fraunhofer region

In this region, the sound pressure is given by the equation :

$$p(r, \theta) = \frac{1}{r} \frac{J_1\left(\frac{\pi D}{\lambda} \sin \theta\right)}{\frac{\pi D}{\lambda} \sin \theta} \quad J_1 \text{ is the Bessel function of first order (réf.) [3]}$$

The energy extends radially with a $\frac{1}{r^2}$ dependance of the sound intensity.

If we define $\alpha = \frac{D \sin \theta}{\lambda} = 3 Df \sin \theta$, D in meters, f in kHz, the pressure only depends on α through a universal function.

When D and f vary, the real range of α is given by

$$0 \leq \theta \leq \frac{\pi}{2} \Rightarrow 0 < \alpha < 3 Df$$

When f increases, being constant D, $\alpha_{max} = 3 Df$ also increases, and this reveals an increasing portion of this universal function (zeros are coming in the physical region).

The first zero of $\frac{J_1(\pi\alpha)}{\pi\alpha}$ is at $\alpha_0 = 1.24$

figure 3

In fig. 3 we represent this universal curve. The secondary lobe at $\alpha = 1.6$ is about 18 dB below the main peak. If we define the $\Delta\Omega$ as -6 dB of the main peak we find :

$$\alpha_{-6} = 0.7 \quad \text{thus} \quad \sin \theta_{-6} = \frac{0.7}{3 Df}$$

The increase of directivity with f is a well known phenomenon that can have undesirable consequences in arraying since large dimensions of arrays will have a great influence on the audio spectrum because of the $(Df)^2$ dependance.

I. 2. b – On axis analytical approach

When integrating equation (2) with M or OZ axis, one gets :

$$p(r) \propto \lambda \sin \left(\frac{\pi}{\lambda} \left(\sqrt{r^2 + \frac{D^2}{4}} - r \right) \right)$$

figure 4

The results of the numerical program are shown on figure 4, in dotted line.

When $r \gg \frac{D}{2}$ we have :

$$p(r) \propto \frac{\pi D^2}{4} \frac{1}{2r}$$

This is the region of spherical waves, that we call the Fraunhofer region, where the sound intensity goes as $\left(\frac{1}{r^2}\right)$ and is independant of the frequency.

When $r < \frac{D}{2}$ we have :

$$p(r) \propto \lambda \sin\left(\frac{\pi D}{2} \frac{D}{\lambda}\right) = \frac{1}{3f} \sin\left(3\frac{\pi D}{2} f\right)$$

this is the near region or Fresnel region where the average sound intensity goes as $\left(\frac{1}{r^2}\right)$ and is independant of the distance.

Between these two regions the pressure drops to zero when :

$$\frac{\pi}{\lambda} \left(\sqrt{r^2 + \frac{D^2}{4}} - r \right) = n\pi \quad n \text{ is an integer. (3)}$$

The farthest zero occurs when $n = 1$, that is :

$$r_1 = \frac{\lambda}{2} \left[\left(\frac{D}{2\lambda} \right)^2 - 1 \right] \quad (4)$$

(f in kHz)
(D, λ in meters)

The border line between Fresnel and Fraunhofer can be defined as a distance $r > r_1$, because then we have no longer interference effects. The source can then be considered as a point source and we are in the Fraunhofer region.

We define the border line quantitatively as the distance where the true pressure is 3 dB below the Fraunhofer asymptotic pressure.

When $r > \frac{D}{2}$ we get :

$$p(r) = \frac{1}{3f} \sin(x) \sim \frac{x}{3f} \left(1 - \frac{x^2}{6}\right) \quad \text{with} \quad x = 3f \frac{\pi D^2}{8r}$$

we get r_{border} from the 3 dB condition which translates into :

$$1 - \frac{x^2}{6} = \frac{1}{\sqrt{2}} \Rightarrow x - 1.4 = 3f \frac{\pi D^2}{8r}$$

$$r_{\text{border}} \sim \frac{3\pi}{81.4} f D^2 = 0.84 f D^2$$

On the other hand, we get from (4) that this border line occurs only when $\lambda = \frac{D}{2}$, for greater wavelengths, the sound field is pure Fraunhofer.

We note that in the Fresnel region, when it exists, the pressure, apart from the large pressure dips, is constant (plane wave).

1.2. c – The Fresnel regions

In order to have a more physical approach of the situation, let us consider it from the point of view of Fresnel. According to Fresnel himself we draw spheres centered on M and the radii of which are $r, r + \frac{\lambda}{2}, \dots, r + n \frac{\lambda}{2}$. This is shown on figure 5.

figure 5

The number N_F of Fresnel zones on the circular source is :

$$N_F = \frac{\sqrt{r^2 + \frac{D^2}{4}} - r}{\lambda/2} = \frac{D^2}{2\lambda \left(\sqrt{r^2 + \frac{D^2}{4}} + r \right)}$$

The alternance of positive and negative zones that are seen from M, are responsible for the pressure dips we described in 1.2. a –. The successive rings cancel each other almost exactly leaving a reduced effective radiating area whose radius is :

$$\sqrt{r^2 + \frac{\lambda^2}{4}} - r$$

Back to the number of Fresnel zones : N_F we see that when z increases or f decreases to the point where $N_F = 1$, this gives the limit of the Fraunhofer region.

$$N_F = \frac{D^2}{2\lambda \left(\sqrt{r^2 + \frac{D^2}{4}} + r \right)} = 1$$

$$r_{\text{border}} = \frac{D^2}{4\lambda} - \frac{1}{4}$$

$$r_{\text{border}} = \frac{3}{4} D^2 f \left(1 - \frac{1}{(3Df)^2} \right) \quad \text{when } f < \frac{1}{3D} \quad \text{there is no Fresnel region.}$$

I. 2. d – The geometric diffraction approach

The half angle opening of the sound intensity in the Fraunhofer region is given by :

$$\sin \theta = \alpha_0 \frac{\lambda}{D} = \frac{\alpha_0}{3Df} \quad \text{with } \alpha_0 = 1.24 \quad (6)$$

figure 6

Figure 6 represents the behaviour of sound propagation. The distance of the border line is given by the intersection point between the main pressure lobe of the Fraunhofer calculation and the plane wavefront of the circular piston :

$$r_{\text{border}} = \frac{D}{2 \tan \theta} = \frac{D \sqrt{1 - \sin^2 \theta}}{2 \sin \theta} \quad \text{thus } r_{\text{border}} = \frac{3}{2} \frac{D^2 f}{\alpha_0} \sqrt{1 - \frac{\alpha_0^2}{9 D^2 f^2}} \quad (7)$$

I. 2. e – Summary of border lines expressions

From the on axis analytical approach (I. 2. b) we obtained :

$$r_{\text{border}} = 0.84 D^2 f$$

From the Fresnel zones approach (I. 2. c) we obtained :

$$r_{\text{border}} = \frac{3}{4} D^2 f - \frac{1}{12f} \quad \text{and} \quad \left\{ \begin{array}{l} \lambda > D \\ \text{or } f < \frac{1}{3D} \end{array} \right\} \quad \begin{array}{l} \text{no Fresnel region} \\ \text{only Fraunhofer} \end{array}$$

At last from the geometric diffraction model we found :

$$r_{\text{border}} = \frac{3}{2} \frac{D^2 f}{\alpha_0} \sqrt{1 - \frac{\alpha_0^2}{9 D^2 f^2}} \quad \text{and} \quad \left\{ \begin{array}{l} \lambda > \frac{D}{\alpha_0} \\ \text{or } f < \frac{\alpha_0}{3D} \end{array} \right\} \quad \begin{array}{l} \text{no Fresnel region} \\ \text{only Fraunhofer} \end{array}$$

This proves to be consistent, and hence we will use the last formula :

Let us remind ourselves that this is the distance beyond which we are in the Fraunhofer region, hence the only region where the directivity concept makes sense.

In the case of a single source the border is very near to the source (usually below one meter) and we don't have to care about the Fresnel region or where the directivity and polar plots are useless. On the contrary, when we are involved with arrays we will see that the border can be located very far away.

I. 3. Single rectangular source

We turn now to the description of the sound field produced by a single rectangular vibrating piston on a plane infinite wall. The work has been done by S.P LIPSHITZ and J. VANDERKOOY [4] for a wire which is very similar to a thin strip behaviour.

Let W and H be respectively the width and the height of the strip, and θ, φ the usual angles in spherical coordinates (fig. 7)

figure 7

I. 3. a – The Fraunhofer region

$$p(r, \theta, \varphi) \propto \frac{1}{r} \frac{\sin \pi \alpha}{\pi \alpha} \frac{\sin \pi \beta}{\pi \beta}$$

We use again universal parameters α and β defined as :

$$\alpha = \frac{H}{\lambda} \sin \theta \sin \varphi = 3Hf \sin \theta \sin \varphi \quad \text{and} \quad \beta = \frac{W}{\lambda} \sin \theta \cos \varphi = 3Wf \sin \theta \cos \varphi$$

figure 8

The universal curve for $\beta = 0$ ($\varphi = \frac{\pi}{2}$) is on figure 8. The zeros are given by

$\alpha = n$. The first zero is $\alpha = 1$ and we recall that for a circular source it is $\alpha = 1.24$.

The -6 dB point of the main peak is at $\alpha = 0.6$ which gives :

$$\text{For } \varphi = \frac{\pi}{2}, \quad \sin \theta_{-6} = \frac{0.6}{3Hf}$$

The second lobe occurs at $\alpha = 1.5$ where the intensity is 13.4 dB below the main peak.

As in the case of a circular piston we have an increase of directivity with the square of the frequency.

I. 3. b – On axis analytical approach

Unlike the circular source we cannot get simple closed forms even if we neglect the $\frac{1 + \cos \psi}{2}$ term.

Assuming that the width is much smaller than the height and smaller than the wavelength at high frequencies, no interference is expected across the width. We are still assuming a

constant volume acceleration (see fig. 9).

figure 9

Supposing $d \geq \frac{H}{2}$:

$$r \sim d + \frac{y^2}{2d}$$

Let us split the integral into real and imaginary parts.

$$P(M) \propto 2 \frac{e^{ikd}}{d} \int_0^{\frac{H}{2}} e^{jk \frac{y^2}{2d}} dy = e^{ikd} \int_0^{v_0} \frac{e^{iv}}{\sqrt{v}} dv$$

$$v_0 = \frac{k}{2d} \left(\frac{H}{2}\right)^2 = \frac{3\pi}{4} f \frac{H^2}{d}$$

$$P(M) \propto \sqrt{\frac{2}{kd}} e^{ikd} \sqrt{2\pi} (f_R + f_I) \begin{cases} f_R(v_0) = \frac{1}{\sqrt{2}} \int_0^{v_0} \frac{\cos t}{\sqrt{\pi t}} dt \\ f_I(v_0) = \frac{1}{\sqrt{2}} \int_0^{v_0} \frac{\sin t}{\sqrt{\pi t}} dt \end{cases}$$

f_R and f_I are well known tabulated Fresnel Integrals. [5]

figure 10

Figure 10 represents the affix of $f_R + if_I$ as v_0 goes from 0 to ∞ . This is to be understood as for a given distance from the source, the complex pressure value is oscillating, following the Cornu spiral, as $H^2 f$ goes from 0 to ∞ .

The sound intensity is proportional to the modulus square of OP.

The sound intensity never goes to zero, and this makes a major difference with the circular source. However the intensity goes to a limit with an oscillatory behaviour of smaller and smaller amplitudes around the average value:

$$\frac{\lambda}{d} = \frac{1}{3df}$$

The maxima and minima of f_R and f_I are given by:

$$f_R \rightarrow v_0 = \left(n - \frac{1}{2}\right) \pi$$

$$f_I \rightarrow v_0 = n\pi \quad n \text{ being an integer.}$$

The dips of P are thus given by an intermediate value that we can approximate as:

$$v_n = \left(2n - \frac{1}{4}\right) \pi = \pi \frac{H^2}{4} \frac{3f}{d_n}$$

$$d_n = \left(\frac{H}{2}\right)^2 \frac{1}{\lambda \left(2n - \frac{1}{4}\right)} = \frac{H^2}{4} \frac{3f}{2n - \frac{1}{4}}$$

The farthest dip is given by:

$$d_1 = \frac{3}{7} H^2 f$$

To be compared with the asymptotic formula (4) changing H in D we obtain the same minimum pressure locations.

We now have an idea of the dependance of the pressure on d when $v_0 \ll 1$ we can approximate:

$$e^{iv} \sim 1 \quad \text{thus} \quad \int_0^{v_0} \frac{dv}{\sqrt{v}} = 2\sqrt{v_0}$$

so that:

$$P(M) = \sqrt{\frac{2}{kd}} e^{ikd} 2 \sqrt{\frac{k}{2d}} \frac{H}{2} = H \frac{e^{ikd}}{d}$$

In this region, the sound intensity goes like $\frac{1}{d^2}$ and is non-frequency dependant: it corresponds to the Fraunhofer region.

As we see on figure 10, the border can be determined by:

$$v_0 = 1.5 \quad n_{\text{border}} = \frac{\pi}{2} H^2 f$$

When d is farther than the border line as defined above, we have a Fraunhofer region with a regular behaviour of the sound intensity which decreases as $\frac{1}{d^2}$, and when d is smaller we have an oscillatory behaviour around an average value of $\frac{1}{d}$.

I. 3. c – Fresnel zones

When compared to the circular piston we have the same pattern of rings (figure 11) but their areas are widely reduced except for the central zone.

figure 11

The area of the central zone is approximately

$$W \sqrt{\left(d + \frac{\lambda}{2}\right)^2 - d^2}$$

when $d \gg \lambda$ the area goes as $\sqrt{\frac{d}{f}}$

The sound intensity is thus proportional to $\frac{1}{df}$ and we find again the Fresnel region typical of a cylindrical wave which decreases as $\frac{1}{d}$ and not as $\frac{1}{d^2}$

Again the number of Fresnel zones, N_F , is given by

$$N_F = \frac{H^2}{2\lambda \left(\sqrt{d^2 + \frac{H^2}{4}} + d \right)}$$

The distance of border line defined as $N_F = 1$ is given by :

$$r_{\text{border}} = \frac{3}{4} H^2 f - \frac{1}{12f} \quad (\text{still } f \text{ in Khz})$$

(d in meters)

I. 3. d – The geometric diffraction approach

From I. 3. a the first zero of the diffraction field is given by $\alpha = 1$ that is for

$$\varphi = \frac{\pi}{2}$$

$$3 H f \sin \theta = 1$$

figure 12

The scope is the same as in figure 6. See figure 12.

And we obtain, from the same calculation

$$r_{\text{border}} = \frac{3}{2} \frac{H^2 f}{\alpha_0} \sqrt{1 - \frac{\alpha_0^2}{9 (Hf)^2}} \quad (\alpha_0 = 1)$$

I. 3. e – Summary of border lines equations

From the on axis analytical approach (I. 3. b) we obtained :

$$r_{\text{border}} = \frac{\pi}{2} H^2 f$$

From the Fresnel zones approach (I. 3. c) we obtained :

$$r_{\text{border}} = \frac{3}{4} H^2 f - \frac{1}{12f} \left\{ \begin{array}{l} \lambda > H \\ \text{or } f < \frac{1}{3H} \end{array} \right\} \quad \text{no Fresnel region}$$

From the geometric diffraction picture (I. 3. d) we obtained :

$$r_{\text{border}} = 1.5 H^2 f \sqrt{1 - \frac{1}{(3Hf)^2}} \left\{ \begin{array}{l} \lambda > H \\ \text{or } f < \frac{1}{3H} \end{array} \right\} \quad \text{no Fresnel region}$$

For convenience, choosing the geometric one :

$$r_{\text{border}} = \frac{3}{2} H^2 f \sqrt{1 - \frac{1}{(3Hf)^2}} \quad r_{\text{border}} \text{ in meters}$$

and $f < \frac{1}{3D}$ no Fresnel region.

I. 4. Comparison between point, circular and strip sources

The comparison of border line expressions I.2.e and I.3.e for circular or rectangular sources, shows that one can sum up these expressions into a unique one if D designates either the diameter of a circular piston or the height of a rectangular piston.

$$r_{\text{border}} = \frac{3}{4} \frac{(D)^2 f}{\alpha_0} \sqrt{1 - \frac{\alpha_0^2}{9 (D)^2 f^2}}$$

with $\alpha_0 = 1.24$ for a circular piston.

$\alpha_0 = 1$ for a strip.

In order to make the comparison we display on figure 13, the sound intensity as a function of the frequency and the distance d .

The point source radiates a pure Fraunhofer field and we can represent it as a piece of cake.

When the source is no longer a point but has a non neglectible size, a part of the "cake" disappears leaving the Fresnel zone.

The circular and the strip show the same border line if $D = H$, but they differ significantly inside the Fresnel region.

| | circular | strip | point |
|-----------------------------|---|---|-----------------|
| average I for a given f | constant inside Fresnel $\frac{1}{r^2}$ in Fraunhofer | $\frac{1}{r}$ in Fresnel $\frac{1}{r^2}$ in Fraunhofer | $\frac{1}{r^2}$ |
| average I for a given r | $\frac{1}{f^2}$ in Fresnel C^m in Fraunhofer | $\frac{1}{f}$ in Fresnel C^m in Fraunhofer | C^m |
| locations of intensity dips | Same locations | | no dips |
| depth of grooves | infinite (cancellations) | a few dB and decreasing | |

These border lines being as such it is impossible to equalize the sound intensity in frequency because as the distance changes, the transition of the intensity between Fresnel and Fraunhofer occurs at various frequencies.

Furthermore the transition line reveals large differences in the propagation mode of the sound wave, from plane wave to spherical wave in the case of a circular piston, and from cylindrical wave to a spherical wave in the case of a strip (fig. 13).

Once again, there should not be matter of concern as long as a single electrodynamic loudspeaker is involved because of its usual range limited to $\lambda > \text{Diameter}$, and also because such a loudspeaker cannot be considered as a real flat piston at high frequencies.

However the problem may occur at high frequencies for ribbon and electrostatic loudspeakers that have large dimensions compared to the wavelength.

At last these phenomenons have major consequences in arrays.

figure 13

II. PLANE ARRAYS

II. 1. Generalities

We restrict the term array to an ensemble of identical elementary sources arranged in a regular network that can be described by a few numerical values that are :

- the size of the elementary piston;
 - D for the diameter of a circular piston
 - H, W respectively the height and width of a rectangular piston
- the network parameters :
 - vertical : N_v sources piled up with a step STEP_v
 - horizontal : N_H sources lined up with a step STEP_H (see figure 14)

figure 14

The step being defined as the distance between the centers of two contiguous sources.

The array is operating in free field conditions.

We developed a program to calculate the pressure produced by $N_v \times N_H$ identical sources, organised in N_H columns, each column making an angle θ_0 with its neighbours. In this work we set $\theta_0 = 0$

The program is based on Fresnel rigorous calculations and Fraunhofer approximation at long distance (see Appendix I and II).

The geometrical diffraction approach depicted in figure 15 reveals a new region : the transition region comprised between the individual Fresnel region of each source and the collective Fraunhofer region of the array. The transition region will be named chaotic as it is not amenable to simple description (at least we did not succeed to do so).

figure 15

Once again, the same calculation as for (I. 2. d -) leads to the expression of two border lines :

- r_{border}^1 is the border line between individual Fraunhofer and chaotic region defined by :

$$r_{\text{border}}^1 = \frac{\text{STEP}}{2 \text{tg } \theta_1}$$

$$\text{with } \sin \theta_1 = \frac{\alpha_0}{3 D f} \quad \alpha_0 = 1.24 \text{ circular piston} \\ = 1 \quad \text{rectangular piston}$$

$$\text{thus } r_{\text{border}}^1 = \frac{3}{2} \frac{D \text{ STEP } f}{\alpha_0} \sqrt{1 - \frac{\alpha_0^2}{9(Df)^2}}$$

D is in this case either the diameter or the height of the strip.

- d_{border}^2 is the border line between chaotic and collective Fraunhofer region defined by :

$$r_{\text{border}}^2 = \frac{N \text{ STEP}}{2 \text{tg } \theta_2}$$

$$\sin \theta_2 = \frac{1}{3 N \text{ STEP } f}$$

$$\text{thus } r_{\text{border}}^2 = \frac{3}{2} N^2 \text{ STEP}^2 f \sqrt{1 - \frac{1}{9(N \text{ STEP } f)^2}}$$

NB : STEP is either STEP_H , or STEP_v , according to the plane of observation, respectively horizontal or vertical.

By analogy with I.3.e, we have the following simplified expressions of the different border lines :

- individual Fresnel /Fraunhofer border line.

$$r_{\text{border}}^0 = \frac{3}{2} \frac{D^2 f}{\alpha_0} - \frac{\alpha_0}{12 f} \quad \text{for } f > \frac{\alpha_0}{3D}$$

- individual / chaotic border line.

$$r_{\text{border}}^1 = \frac{3}{2} \frac{D \text{ STEP } f}{\alpha_0} - \alpha_0 \frac{\text{STEP}}{9 D f} \quad \text{for } f > \frac{\alpha_0}{3D}$$

D is either the diameter or the height of the piston $\alpha_0 = 1$ (rectangular) ; $\alpha_0 = 1.24$ (circular).

- chaotic / collective Fraunhofer border line

$$r_{\text{border}}^2 = \frac{3}{2} N^2 \text{ STEP}^2 f - \frac{1}{12 f} \quad \text{for } f > \frac{1}{3 N \text{ STEP}}$$

figure 16 illustrates the partition in the plane (d, f).

figure 16

II. 2. Description of the Fraunhofer region for : ray

II. 2. a – The form factor

Let p_1 be the pressure field of one source at point M. The pressure delivered by N identical sources can be written as the product of the individual pressure of one source by a form factor $F_N(\theta, \varphi)$: [6] (see fig. A2)

$$P_N(r, \theta, \varphi) = p_1(r, \theta, \varphi) F_N(\theta, \varphi)$$

F_N depends only upon the positions of the sources. It is a geometrical quantity

$$F_N(\theta, \varphi) = \sum_{i=1}^N e^{ik \vec{r}_i \cdot \vec{\delta n}}$$

$\vec{\delta n}$: displacement vector of the i th source, one of them being chosen as origin.
For a wall of regularly spaced boxes we have :

$$F_N(\theta, \varphi) = \frac{\sin\left(N_V \frac{k \text{ STEP}_V}{2} \sin\theta \sin\varphi\right)}{\sin\left(\frac{k \text{ STEP}_V}{2} \sin\theta \sin\varphi\right)} \cdot \frac{\sin\left(N_H \frac{k \text{ STEP}_H}{2} \sin\theta \cos\varphi\right)}{\sin\left(\frac{k \text{ STEP}_H}{2} \sin\theta \cos\varphi\right)}$$

$$N = N_V N_H$$

We will observe F_N when $N_H = 1$ because it does not change the fundamental understandings.

We define the parameter :

$$\alpha = \frac{\text{STEP}}{\lambda} \sin\theta \sin\varphi = 3 \text{ STEP } f \sin\theta \sin\varphi$$

Under such conditions :

$$F_N = \frac{\sin(N\alpha\pi)}{\sin\alpha\pi}$$

figure 17 et 18

The square of this function is represented in figures 17, 18 for $N = 4$ and 16.

For $\theta = 0$ we have a reinforcement N^2 . The axial intensity is thus proportional to N^2 , but

the width of peaks is reduced by $\frac{1}{N}$; the energy is thus conserved.

The full Width at half maximum (-6 dB point) occurs at $\alpha_0 = \frac{0.6}{N}$, but each time that α is an integer the sound intensity goes up to N^2 , the value at $\theta=0$. Those secondary peaks that can be far from axis must be carefully looked after. Because of these peaks, the usual concept of directivity may be irrelevant.

The number of smaller peaks between two N^2 peaks is $N - 2$ and the intensity of the second peak is always in the ratio :

$$\frac{4}{9\pi} = 0.044$$

or -13.5 dB and is non-dependant of N.

II.2.b- Limit conditions for existence of secondary lobe

We must be able to know where is the real region.

$$\alpha = \frac{\text{STEP}}{\lambda} \sin\theta \sin\varphi = 3 \text{ STEP } f \sin\theta \sin\varphi$$

α_{max} is given by $\theta = \varphi = \frac{\pi}{2}$ that is

$$\alpha_{\text{max}} = \frac{\text{STEP}}{\lambda} = 3 \text{ STEP } f$$

The second N^2 peak enters the physical region as soon as :

$\alpha_{\text{max}} > 1$ that is $3 \text{ STEP } f > 1$

The form factor may thus be considered as function of α and N, limited to the real region $\alpha \in [0, 3 \text{ STEP } f]$ and depending only of the number of sources.

II.2.c- Calculation of the sound pressure

The resultant pressure is the product of F_N by the individual pressure p_1 .

circular :

$$p_1 = \frac{J_1\left(\pi \frac{D}{\lambda} \sin\theta\right)}{\frac{\pi D}{\lambda} \sin\theta}$$

slot :

$$p_1 = \frac{\sin\left(\pi \frac{H}{\lambda} \sin\theta\right)}{\frac{\pi H}{\lambda} \sin\theta}$$

As for the form factor F_N , the real region depends on D (or H) and f.

$$\begin{aligned} \text{circular : } \alpha_{\text{max}} &= 3 D f \\ \text{slot : } \alpha_{\text{max}} &= 3 H f \end{aligned}$$

The figure 2 of appendix II shows the sound intensity produced by a line array of 16 circular pistons, that is the product of p_1 by F_N after scale normalization.

II. 3. Chaotic region

II. 3. a. Location in the distance versus frequency plane

To have an idea of the complexity of the chaotic region, we can make use of the propagation equation for the sound pressure p :

$$\Delta p + k^2 p = 0$$

For an infinite number of elementary sources, all identical, and if we restrict to a bidimensional problem, we can separate the pressure into two terms :

$$p(x, z) \propto f(z) \sum_n a_n e^{ik_s x}$$

$$\text{with } k_s = \frac{2\pi}{\text{STEP}}$$

The individual harmonics verify :

$$\frac{d^2 f}{dz^2} = (n^2 k_s^2 - k^2) f(z)$$

If we suppose that $f(z) \propto e^{\frac{z}{L}}$, we obtain :

$$\frac{1}{L} = \frac{2\pi}{\lambda} \sqrt{1 - \left(\frac{n\lambda}{\text{STEP}}\right)^2}$$

When, $\lambda > \frac{\text{STEP}}{n}$ the n^{th} spatial harmonic propagates and when $\lambda < \frac{\text{STEP}}{n}$ we have a strong exponential attenuation of the network structure perturbations. As an example when STEP is about one meter, the chaotic region appears for any frequency higher than 300 Hz. We obtain the same condition as in II. 2. b.

$$f < 1/3 \text{ STEP}$$

II. 3. b. Rough description of the sound pressure

A means to evaluate the huge differences between discrete sound sources and a uniform one, when $f > \frac{1}{3 \text{ STEP}}$, consists in considering point sources (D or $H = 0$) in the Fraunhofer region. To be specific, let us consider, on figure 19, five punctual sources and let us evaluate the sound pressure at M .

Figure 19

Staying at distances $d \geq \frac{N_v \text{ STEP}}{2}$ we can approximate r_n as :

$$r_n = d \left(1 + \frac{n^2 \text{ STEP}^2}{2d^2} \right)$$

Assuming, as usual $\frac{1 + \cos \psi}{2} \sim 1$, have :

$$p(M) \sim \frac{e^{ikd}}{d} + \frac{2}{d} e^{ikd} \sum_1^{\frac{N_v \text{ STEP}}{2}} e^{ikn^2 \frac{\text{STEP}^2}{2d}}$$

$$p(M) \sim \frac{e^{ikd}}{d} N_v \left[\frac{1 + 2 \sum_1^{\frac{N_v \text{ STEP}}{2}} e^{ikn^2 \frac{\text{STEP}^2}{2d}}}{N_v} \right] = (\text{Fraunhofer} * \text{Formfactor})$$

figure 20

Figure 20 shows the affix of the form factor term, when $N_v = 5$.

The modulus of the form factor is OP, where P is the point whose position depends upon $\varphi = \pi \frac{\text{STEP}^2}{\lambda d}$.

We see that as d or f varies, the length OP varies constantly by large amounts. The amplitude of the oscillations range from

$(0.2)^2 = 4.10^{-2}$ to $1^2 = 1$. This is 14 dB. The dip positions are given by :

$$d_n = \frac{3}{2n+1} \frac{\text{STEP}^2}{\lambda} = \frac{9 f \text{ STEP}^2}{2n+1}$$

By comparison, a continuous "ribbon" of jointed strips of height STEP, considered as a single slot of height $H = N_v \text{ STEP}$ and strength $\frac{1}{\text{STEP}}$ has been studied in I.3.

The 5 point sources always have a $\frac{1}{d^2}$ decreasing sound intensity. This is in contrast with the single strip which decreases as $\frac{1}{d}$ in the Fresnel region.

Furthermore the oscillations of the intensity are always minimized in the case of the strips.

II. 4. The Fresnel region

Its location is shown on figure 16. The sound pressure has exactly the same behaviour than the single sources described in § 1.

A flat wall generates a plane wave, whose zeros move along the same lines in (d, f) as in I. The on-axis pressure decreases to zero as in the case of circular individual sources.

In the case of a single column, i.e $N_{11} = 1$, it generates cylindrical waves expanding as

$$p(d) \propto \frac{1}{\sqrt{d}}$$

Hence, when $f < \frac{1}{3 \text{ STEP}}$, the Fresnel approach is correct and the discrete network of sound sources is equivalent to a continuous extended sound source.

II. 5. Arrayability condition for a plane array

One can picture an array as the sum of two virtual sources A and B, respectively producing a pressure P_A and P_B .

A : is the continuous ideal source whose size is the size of the array.

B : is the network of elementary single sources formed by the space between actual sources, vibrating in phase opposition to A. It is the perturbation to A.

The resultant pressure in the sum of the two terms $P = P_A + P_B$.

P_A has been described in I and P_B in II.

According to equation (2), the maximum of P_A is obtained on axis and is proportional to the height of the array : $(N - 1) \text{ STEP} + D$ (7).

P_B is mainly governed by the form factor, this being magnified when the space between actual sources decreases. The secondary N^2 peak of P_B can be responsible for an important off axis secondary lobe.

It occurs at $\sin \theta_c = 1/3 f \text{ STEP}$.

We define a condition for arrayability as being :

1. absence of chaotic zone ;
2. no peak outside of the main lobe shall be higher than - 12 dB below the main peak pressure.

When $f < 1/3 \text{ STEP}$, this peak does not enter the physical region.

When $f > 1/3 \text{ STEP}$, the pressure of the secondary peak is proportional to the effective size multiplied by $\sin \pi \alpha / \pi \alpha$ (see I.3.a) that is

$$(N - 1) (\text{STEP} - D) \frac{\sin \pi (1 - D/\text{STEP})}{\pi (1 - D/\text{STEP})} \quad (8)$$

Requiring the total pressure at $\theta = \theta_c$ not to be lower than - 12 dB below the maximum on axis pressure, leads to a very simple limit condition, derived from (7) and (8), that is : $D/\text{STEP} > 0.80$ (9).

This restrictive condition can be achieved in the case of a rectangular source ; but it will never be the case of a circular source even when the sources are close together.

This is related to the average value of the size of a circle of diameter D, that is

$$\langle D \rangle = 2D/\pi.$$

In this case $\langle D \rangle / \text{STEP}$ is always below $2/\pi = 0.64$ even when $D = \text{STEP}$ and condition (9) is never achieved.

Returning to figure 16, which shows an absolute limit in frequency, $f < 1/3 \text{ STEP}$ for any D, to match a good arrayability defined in II.3.a as the absence of chaotic region, this reveals now another condition : the size of the source with respect to the network step, which can be interpreted as a « filling coefficient» η giving :

$$\eta = D/\text{STEP} > 0.8 \text{ for any frequency.}$$

Reciprocally the two conditions for arrayability in a plane defined previously are met as we obtain

either : $f < 1/3 \text{ STEP}$ for any D :

or : $\eta = D/\text{STEP} > 0.8$ for any f.

III. CONCLUSION

This paper sets forward the basic difference in the sound field structure produced by a single source and a plane array. The existence of a chaotic region and the form factor in arrays is responsible for large irregularities in the sound intensity and spatial coverage of the audience zone, which also means that a usual concept like directivity is irrelevant in this case.

The only mean to solve these problems is to define criteria for arrayability, which, when achieved, allow the array to behave exactly like an equivalent single source having the same size as the array.

We have seen that these criteria are very strict, the first one, $f < 1/3 \text{ STEP}$ for any D leads to a network step of less than 3.3 cm when a 10 kHz upper frequency limit is to be achieved with punctual sources ; the second one, leads to a «filling coefficient» of more than 80 % of the array with flat isophase rectangular pistons.

If one of these two conditions is achieved, the network can be considered as arrayable. The chaotic region disappears, leaving only the Fresnel and Fraunhofer regions, which characterize the single source.

The main results are shown in figures 13 and 16, where we draw the average behavior of the sound intensity and the border lines in the distance frequency plane.

For a single loudspeaker, the Fresnel region never enters the audience zone, the border line being below 1 meter.

When considering arrays 3 meters high, the border line for 1 kHz is 15 meters away from the array and for 10 kHz, it is 150 meters !!!!

This shows the importance of the Fresnel zone in arrays. Once again usual concepts like $1/r^2$ intensity dependence, directivity and polar plots are irrelevant in that region, these concepts being only relevant in the Fraunhofer region, which can be very far from the array.

APPENDIX I

Description of the exact treatment of circular piston software.

This program has been written to examine the Fresnel and chaotic regions of a sound field radiated by arrays of circular pistons.

Using formula I, hence keeping the term $\frac{1 + \cos \psi}{2}$, we split the area of each circular piston in polar areas as shown in figure A I.

figure A I

Then, we approximate the integral of formula I by a discrete sum of these surface elements.

$$p(M) \propto \sum_{r \leq r_0} \frac{e^{ikr}}{r} \frac{1 + \cos \psi}{2} \rho d\rho d\theta$$

$$k = \frac{2\pi}{\lambda} \quad \text{and} \quad r = \sqrt{M_i M^2}$$

ψ is the angle between $\vec{M_i M}$ and the normal to the piston under consideration.

Then the sound intensity is proportional to the squared modulus of the complex pressure $p(M)$.

APPENDIX II

Description of the curved array simulation software.

The program calculates the sound intensity delivered by N_{11} columns of N_v sources each. Each column is tilted by an angle θ_0 with respect to the contiguous ones. The origin is taken on the first column at its center.

We are in the Fraunhofer region, for each column, using the analytical expression with the form factor of paragraph II.2.a.

The figure A II shows the different variables.

Figure A II

Let be :

O_n : the center of the n^{th} column.

\hat{r} : the unit vector which goes from the origin O_1 to M the observation point.

$$\hat{r} = \begin{Bmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{Bmatrix} = \sin \theta \cos \varphi \hat{i} + \sin \theta \sin \varphi \hat{j} + \cos \theta \hat{k}$$

$\hat{i}, \hat{j}, \hat{k}$ unit vectors along $O_1 x, O_1 y$ and $O_1 z$.

$p(\theta_n, \varphi_n)$: the pressure field given by the n^{th} column.

The polar angles θ_n and φ_n define the direction of observation with respect to the n^{th} column which is $O_1 xyz$ rotated around $O_1 y$ by the angle $(n-1)\theta_0$.

We have :

$$\widehat{O_1 O_n} = \begin{cases} \text{STEP} \cos\left(\frac{n-1}{2}\theta_0\right) \begin{bmatrix} \sin \frac{n\theta_0}{2} \\ \sin \frac{\theta_0}{2} \end{bmatrix} \cos \frac{n-1}{2}\theta_0 \\ - \text{STEP} \sin\left(\frac{n-1}{2}\theta_0\right) \begin{bmatrix} \sin \frac{n\theta_0}{2} \\ \sin \frac{\theta_0}{2} \end{bmatrix} \cos \frac{n-1}{2}\theta_0 \end{cases}$$

θ_n and φ_n are computed from :

$$\sin \theta_n \cos \varphi_n = \sin \theta \cos \varphi \cos (n-1)\theta_0 + \sin \theta \sin \varphi \sin (n-1)\theta_0$$

$$\sin \theta_n \sin \varphi_n = \sin \theta \sin \varphi$$

$$\cos \theta_n = \cos \theta \cos \varphi \sin (n-1)\theta_0 \cos (n-1)\theta_0$$

The program computes the complex pressure :

$$p_M(\theta, \varphi) \propto \sum_{n=1}^{n=N_H} e^{ik_n \cdot \alpha \alpha_n} p(\theta_n, \varphi_n)$$

Either for a circular or a rectangular source.

For instance for a strip we have :

$$p(\theta_n, \varphi_n) \propto \frac{\sin\left(k\frac{H}{2}\sin\theta_n \cos\varphi_n\right)}{k\frac{H}{2}\sin\theta_n \cos\varphi_n} \cdot \frac{\sin\left(k\frac{W}{2}\sin\theta_n \sin\varphi_n\right)}{k\frac{W}{2}\sin\theta_n \sin\varphi_n}$$

H is the height of the strip and w the width.

The sound intensity is proportionnal to the modulus squared of $p_M(\theta, \varphi)$ and is plotted as a function of the horizontal and vertical distance either as a bidimensional plot or as a projection along the horizontal or vertical axis. (Fif. A2 - 2)

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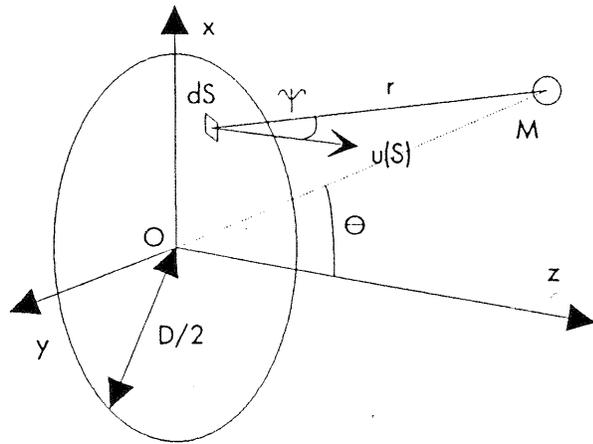


Fig 1. Definition of the variables used in the text.

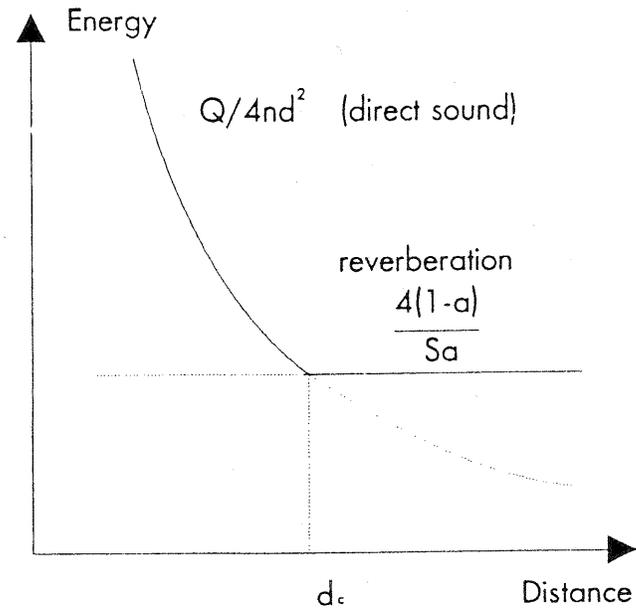


Fig 2. Definition of critical distance.

Fig.4 : Comparison between the on-axis sound intensity , as a function of frequency , for a rigorous derivation (dotted line) and Fraunhofer approximation

Piston diameter : 0.200 m
 Frequency : 20 kHz
 Humidity : 30%
 Absorption : 0
 Discretization / radius step number : 10
 angular step : 10

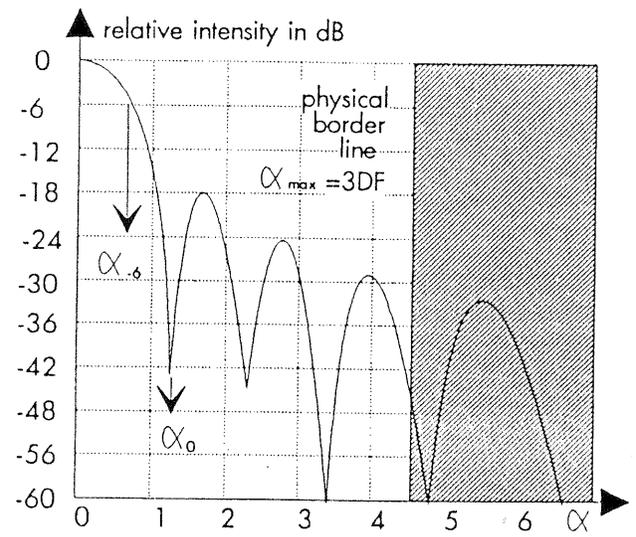
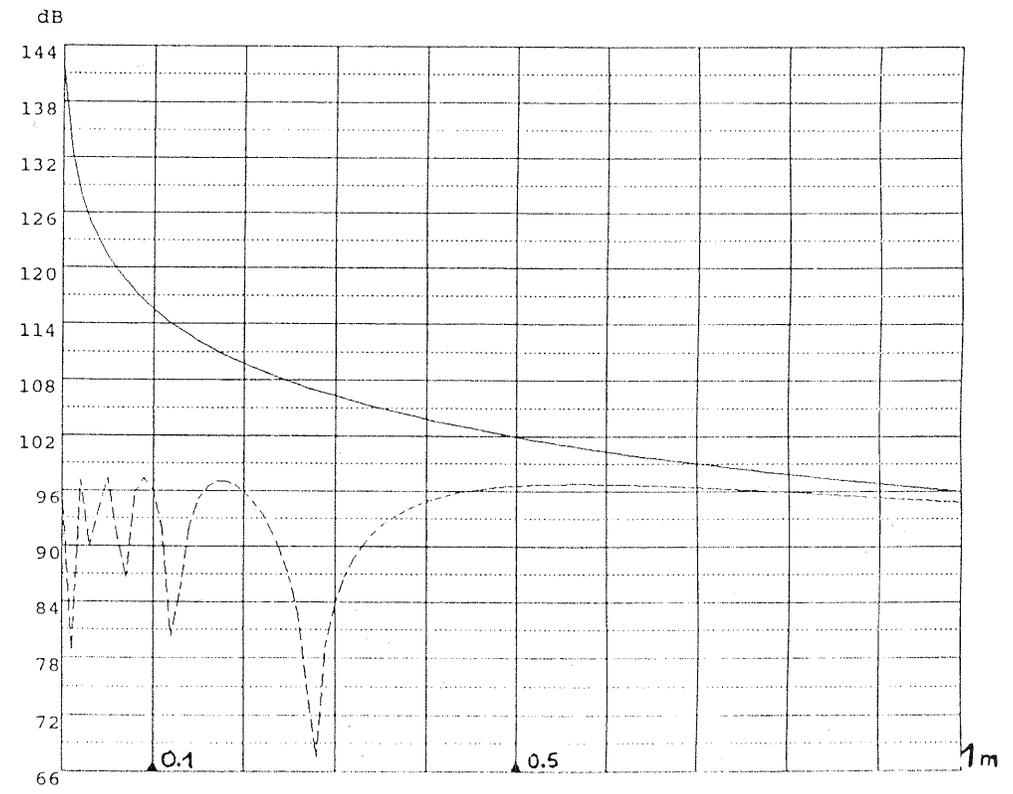


Fig 3. Universal curve for diffraction for a circular piston



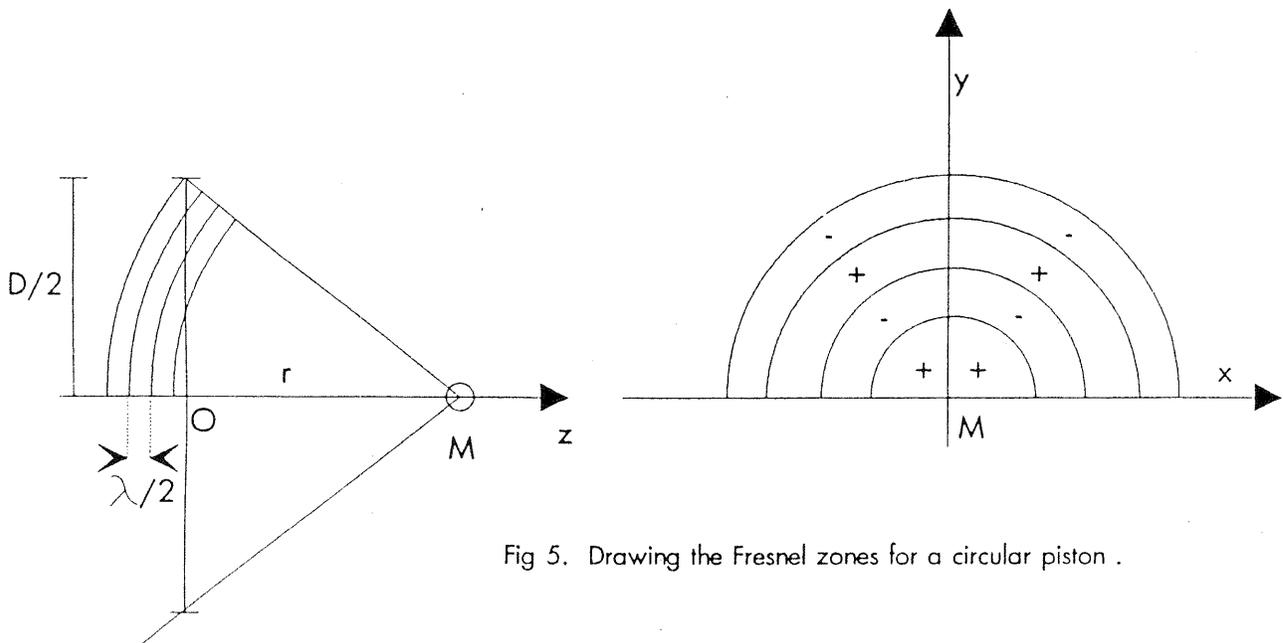


Fig 5. Drawing the Fresnel zones for a circular piston .

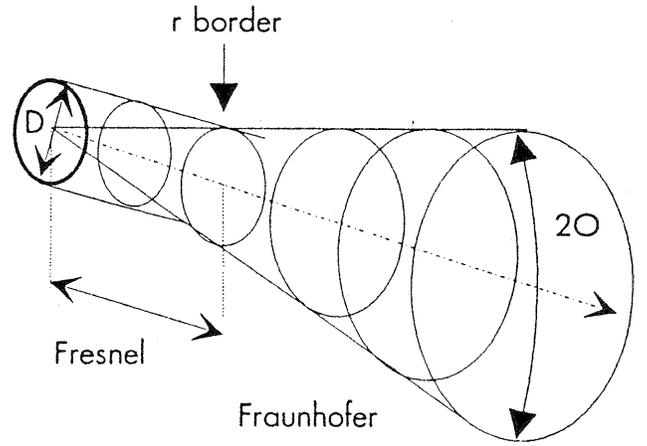


Fig 6. Geometric picture of the Fresnel and Fraunhofer regions .

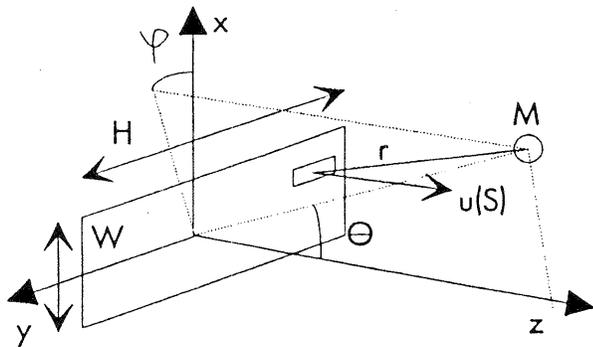


Fig 7. Definition of the variables used in the text.

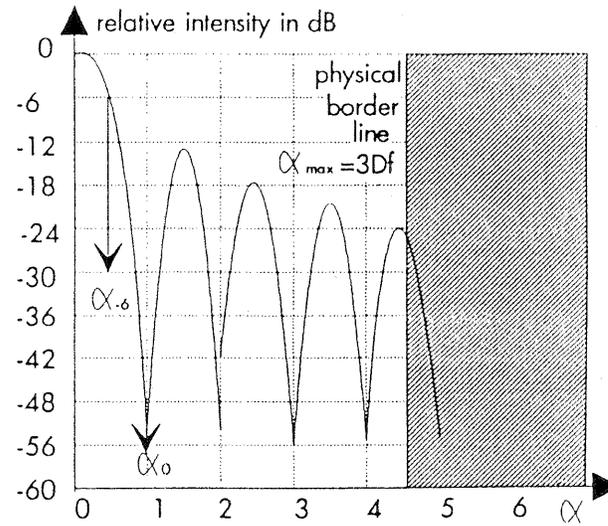


Fig 8. Universal curve for diffraction.
of a rectangular piston.

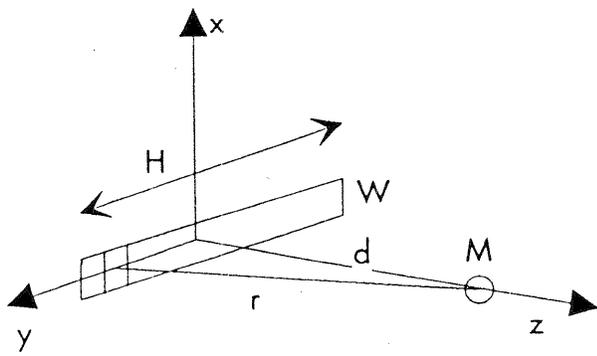


Fig 9. On axis analytic approach .

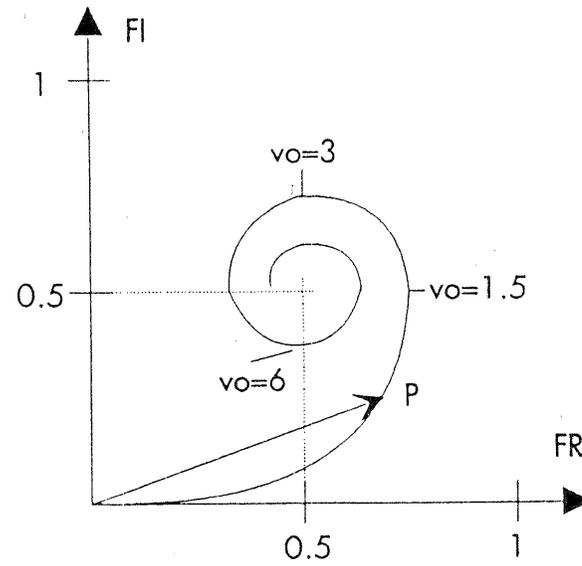


Fig 10. Representation of the Cornu spiral , on axis calculation .

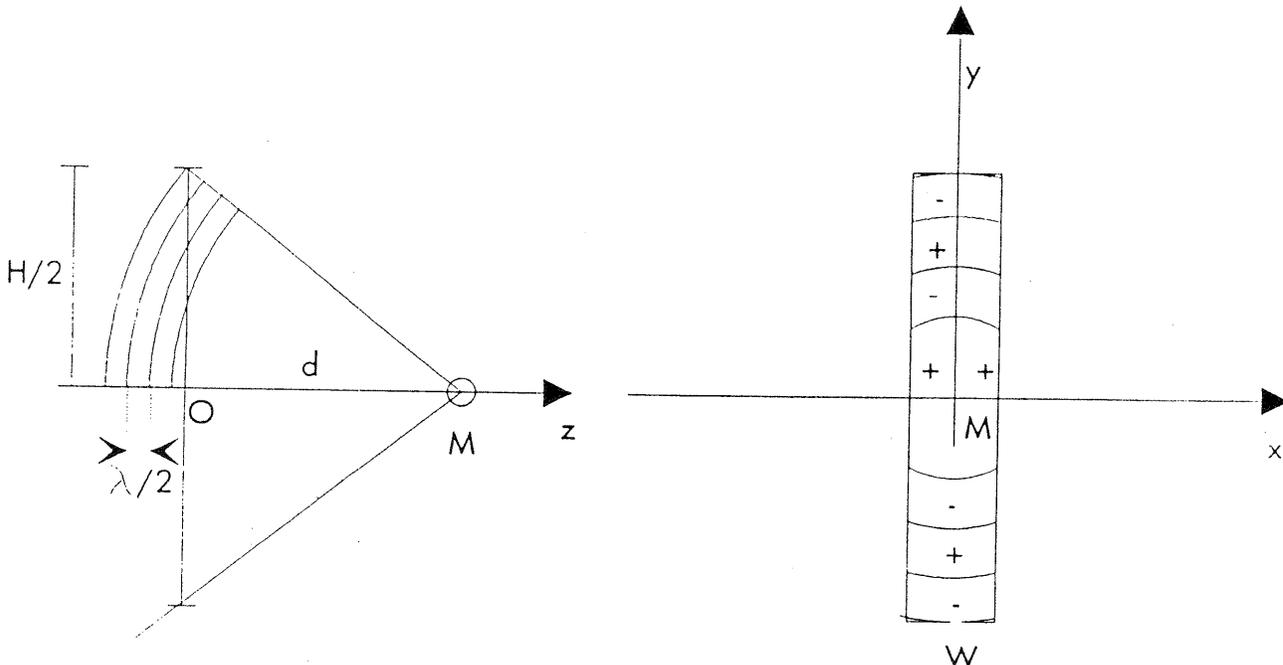


Fig 11. Drawing the Fresnel zones for a slot.

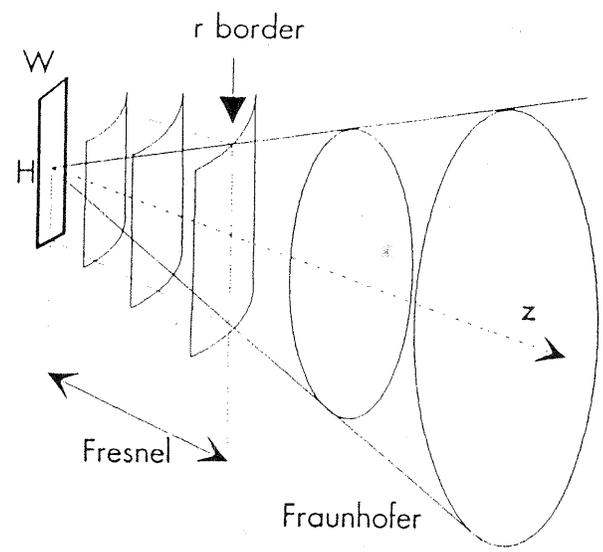


Fig 12. Geometric picture of the Fresnel and Fraunhofer region

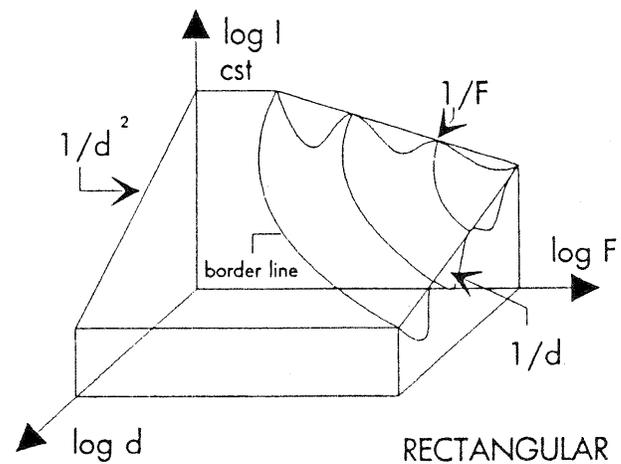
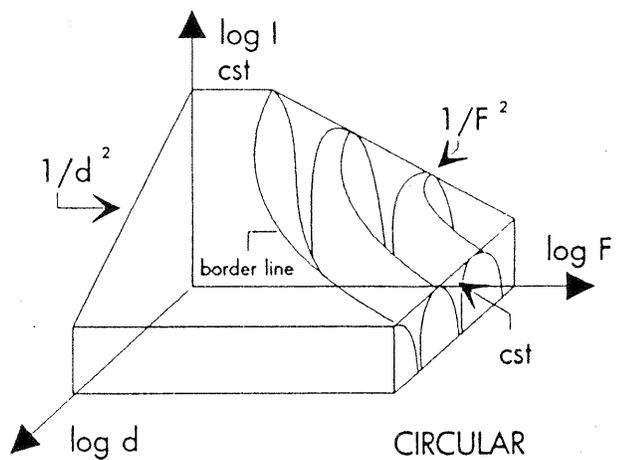
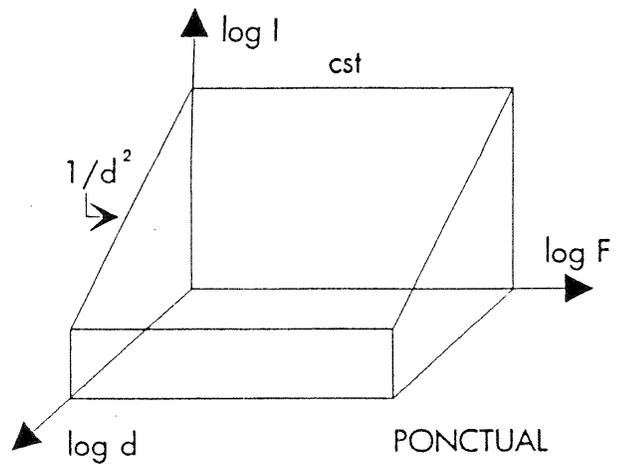


Fig 13. Comparison between sound field radiated by a punctual, a circular and a slot source .

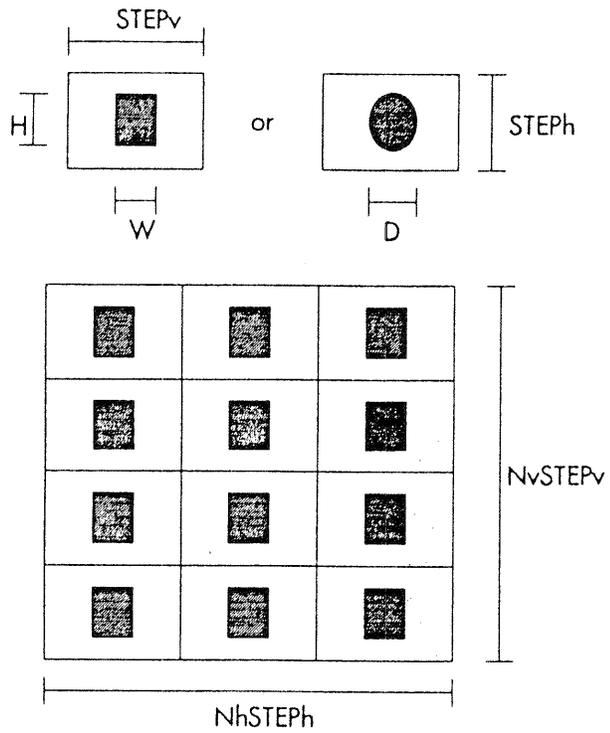


Fig 14. Plane array of identical elementary sources.

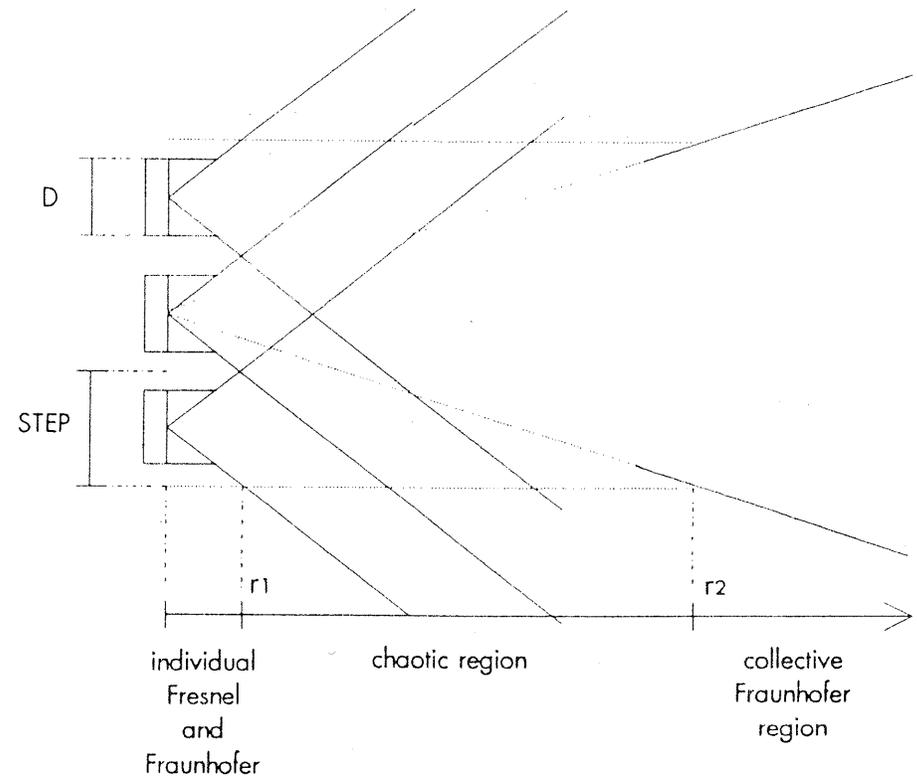


Fig 15. Visualisation of the three regions for arrays .

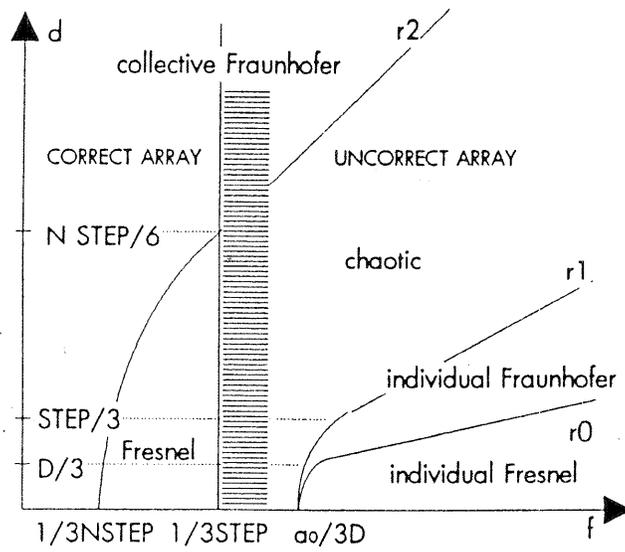


Fig 16. Border lines in the plane (d,f)

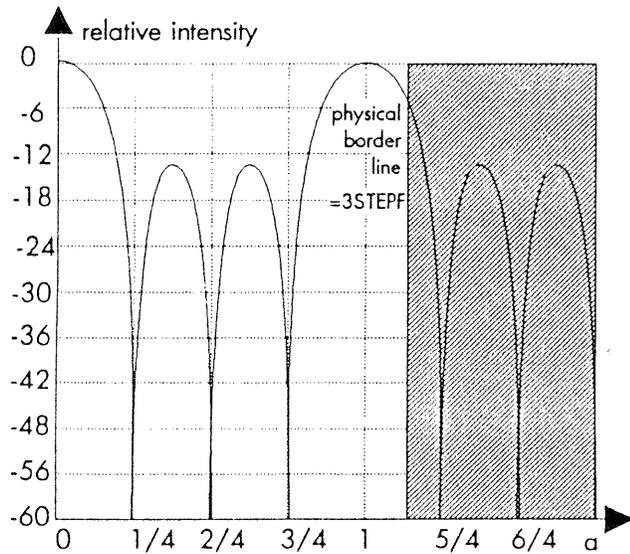


Fig 17. Form factor of 4 sound sources .

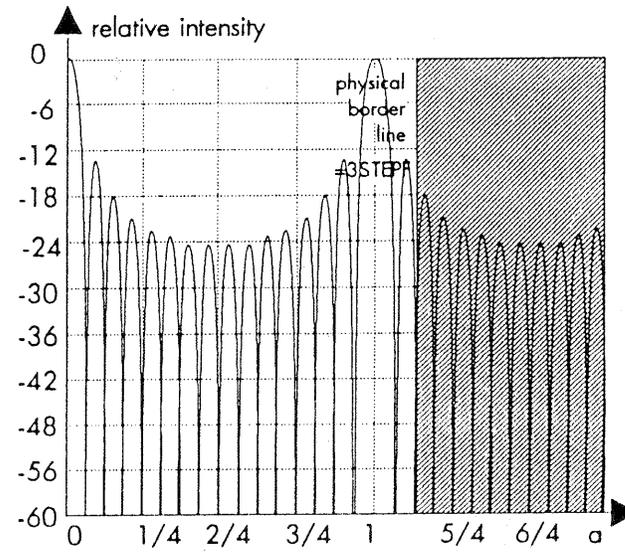


Fig 18. Form factor of 16 sound sources .

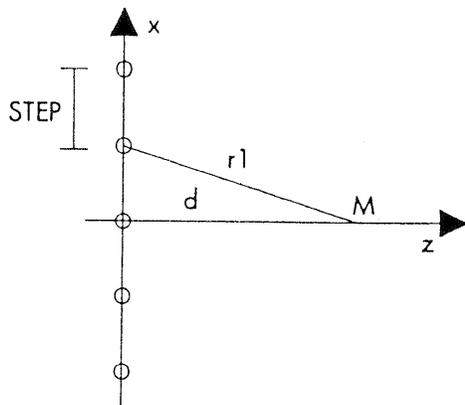


Fig 19. Line array of 5 punctual sources : parameters

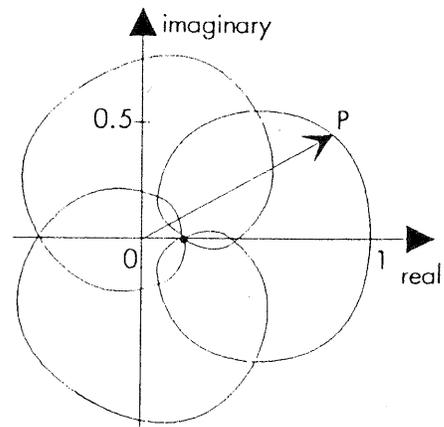


Fig 20. Affix of the form factor for $N_v=5$.

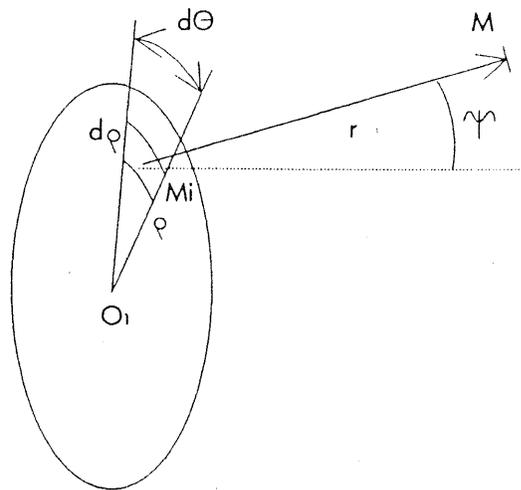


Fig A1. Definition of parameters .

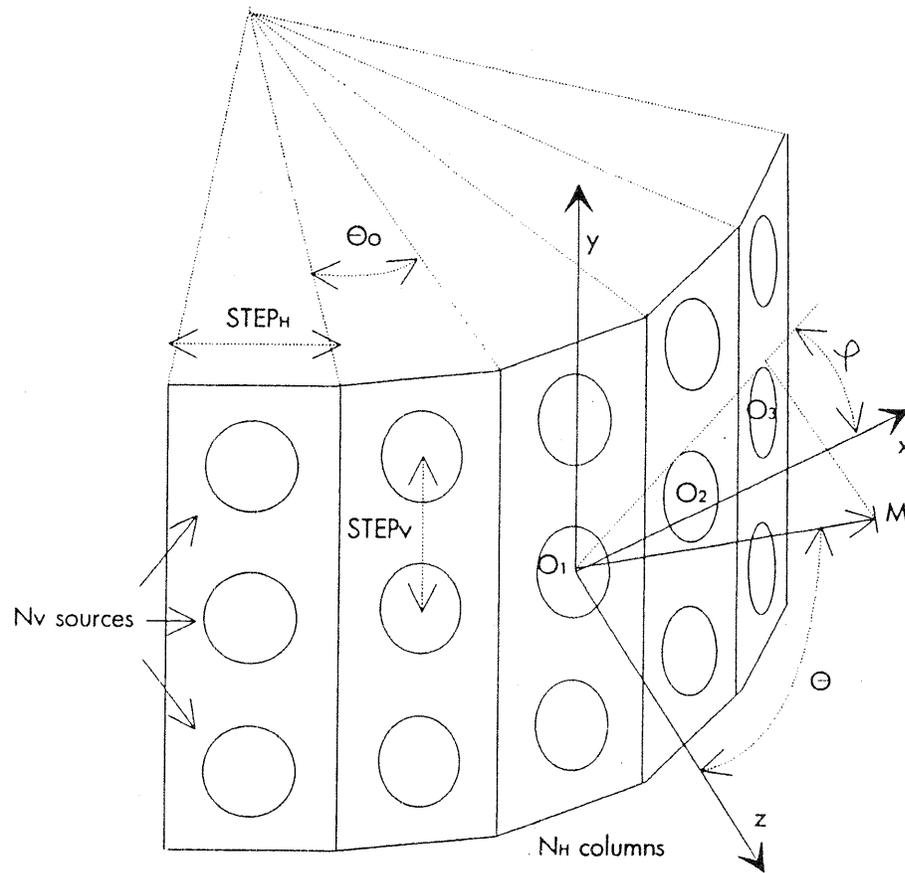


Fig A2. Definition of parameters for curved array .

Fig.A2-2 : Angular distribution of acoustical pressure of a plane array in a Fraunhofer mode.

Piston diameter : 0.2 m
Frequency : 3000 Hz
Humidity : 30%
Absorption : 0
Volume velocity : 1 m**3/s
Nh (columns) : 8
Nv (lines) : 1
Theta0 : 0°

